# Do Zero-Energy Solutions of Maxwell Equations Have the Physical Origin Suggested by A. E. Chubykalo? [Comment on the paper in Mod. Phys. Lett. A13 (1998) 2139-2146] *

**Valeri V. Dvoeglazov**

*Escuela de Física, Universidad Autónoma de Zacatecas*

*Apartado Postal C-580, Zacatecas 98068 Zac., México*

*E-mail: valeri@ahobon.reduaz.mx*

*URL: http://ahobon.reduaz.mx/˜ valeri/valeri.htm*

(August 17, 1999)

Zero-energy solutions of Maxwell's equations have been discovered and re-discovered many times in this century. In this comment we show that the paper by A. E. Chubykalo (which, in fact, also comments on the recent papers) did not explain physical interpretation of this solution and it used doubtful (up-to-date) postulates. Different explanations of the considered problem are possible.

The dynamical free-space Maxwell equations can be written in the form

$$\boldsymbol{\nabla} \times [\mathbf{E} - i\mathbf{B}] + i(\partial/\partial t)[\mathbf{E} - i\mathbf{B}] = 0\,, \tag{1a}$$

$$\boldsymbol{\nabla} \times [\mathbf{E} + i\mathbf{B}] - i(\partial/\partial t)[\mathbf{E} + i\mathbf{B}] = 0\,. \tag{1b}$$

(cf., e.g., formulas (4.19)-(4.22) in ref. [1]). The paper [2] begins with the statement that this set support *non-trivial* solutions with energy $p_0 = \pm|\mathbf{p}|$ and $p_0 = 0$, and refers in this connection to the works [3,6], where the equations (1) have been written for *3-vectors* $\phi_{L,R}(\mathbf{p})$ as opposed to the *fields* in (1a,b).[1] Then, trying to explain the negative-energy and zero-

*Submitted to "Modern Physics Letters A"

[1] The signs in the formulas (1) of ref. [6a], (1,2) of ref. [2] and (20,21) of ref. [6b] are interchanged. No explanations present therein.

energy solutions Chubykalo obtained a new form of the energy density of the electromagnetic field [2, Eq.(28)]. It is based on his instantaneous-action-at-a-distance electrodynamics which includes *implicit* dependence of the fields on time.

In the present note we analyze his work and show that it does not contain sound physical explanation. The main postulate is also required solid experimental and mathematical basis. Nevertheless, we agree that the question of *non-transverse* solutions [8,10,13–17] requires further research.

First of all, I want to mention that the so-called Oppenheimer-Ahluwalia $E = 0$ solutions[2] was discovered by many researchers, whose works [3–8] should be paid credits to.[3,4] They have been analyzed further in [9,10,12].[5] A solution of the similar nature in two-particle system was also considered, see, for example, ref. [11].

A. Chubykalo [2] considers Landau and Lifshitz [18, §31] derivation of the Poynting theorem in the Section I. He objects it on the basis that it does not provide zero energy (negative-energies as well) unless the fields $\mathbf{E}$ and $\mathbf{B}$ are equal to zero. Furthermore, in the Landau-Lifshitz procedure "one implicitly neglects a radiation field which can go off to infinity" (in the opinion of the author of [2]).[6] Chubykalo then divides the electromagnetic

---

[2]Here we use the terminology of the commented paper.

[3]Beforehand, I apologize before those discoverers of the non-transverse solutions of the Maxwell's equations whom I did not mention here. I am sure that a lot of papers remains unnoticed.

[4]Very unfortunately, neither Ahluwalia (the first papers are in ref. [6]) nor Chubykalo papers contain sufficient list of references. I informed Dr. Ahluwalia about previous related works as earlier as in 1994-95.

[5]These papers proved that $E = 0$ solution is not a varying solution in time and it may be *unbounded*.

[6]Indeed, authors of different textbooks on classical/quantum electrodynamics (e. g., ref. [22, p.105]) also assumed the postulate that the fields, their derivatives and Lagrangian, all tend to

fields into *two* parts (which depend on time explicitly and implicitly), writes corresponding Maxwell's equations[7] and, in fact, he **postulates** that some part of the total electromagnetic field ($\mathbf{E}^*$ and $\mathbf{H}^*$ in the notation of [2] to be precise) does **not vanish** at the spatial infinity! See the formula (26) therein. In such a way he **re-normalizes** the energy density subtracting the quantity $\frac{\mathbf{E}^{*\,2}+\mathbf{H}^{*\,2}}{8\pi}$ from the energy density used by Landau *et al.*, see formulas (25)-(28) in [2]. The Chubykalo final result for energy density $\omega$ is

$$\omega = \frac{2\mathbf{E}^* \cdot \mathbf{E}_0 + 2\mathbf{H}^* \cdot \mathbf{H}_0 + E_0^2 + H_0^2}{8\pi} = \frac{\mathbf{E}_{tot}^2 + \mathbf{H}_{tot}^2}{8\pi} - \frac{\mathbf{E}^{*\,2} + \mathbf{H}^{*\,2}}{8\pi}\,. \tag{2}$$

The main problem with the Chubykalo derivation is the following: the integrals are **divergent** when they extend over all the space. This refers to the first term of (25), the integrals in (26) and the dot products in the numerator of the first term of (27). This is easily proven on observing that if $\mathbf{E}^*$ and $\mathbf{H}^*$ do not tend to zero at infinity, they have the asymptotic behavior of $O(r^\alpha)$, $\alpha \geq 0$. In the mean time, $\mathbf{E}_0$ (for instance) refers to Coulomb (Coulomb-like) electromagnetic field and, thus, has the asymptotic behavior $O(1/r^2)$. The factor $r^2$ in the volume element cancels $1/r^2$, leading to the total asymptotic of the integrated expression $O(1), r \to \infty$ in the better case. I am not going to discuss the mathematical validity of subtracting infinite expressions in this case, leaving this problem to specialists. But, to the best of my knowledge, some persons claimed that such procedures do not have sound mathematical foundation [21]. Furthermore, even if one accepts its validity the total energy resulting from integration of (28) over the whole space is to be infinite!? It was noted by Barut [22, p.105] that in the case of non-vanishing fields at the spatial infinity **we cannot**

---

zero at spatial infinity.

[7] In my opinion, equations (20,21) of ref. [2] are just another form of the Maxwell equations for this particular case, in the sense that there is no new physical content if one expects that the Maxwell electrodynamics describes also the Coulomb interaction. Brownstein in [19] showed that, in fact, confusions sometimes arise when physicists use the same symbol $\partial/\partial x^\mu$ for both partial and total derivative. Unfortunately, Brownstein did not cite the previous works on the subject [20].

**expect to find globally conserved quantities**.

In the mean time, the definitional problems of the 4-momentum of the electromagnetic field and the problem of the compatibility of the energy-momentum definition and the Poynting theorem (the formula (10) of [2]) with the Relativity Theory (SRT) are the old ones [24,25]. Pauli [24] stated: "We therefore see that *the Maxwell-Lorentz electrodynamics is quite incompatible with the existence of charges, unless it is supplemented by extraneous theoretical concepts*." But, Rohrlich [26] and Butler [27] seem to have solved this problem establishing for the steady state (when $\mathbf{B} = \mathbf{v} \times \mathbf{E}$ is valid) the new expressions for the energy and momentum of the electromagnetic system

$$P^0 = \gamma \int \left[ (\mathbf{E}^2 - \mathbf{B}^2)/8\pi \right] d^3\sigma = \gamma m, \tag{3a}$$

$$\mathbf{P} = \gamma \mathbf{v} \int \left[ (\mathbf{E}^2 - \mathbf{B}^2)/8\pi \right] d^3\sigma = \gamma \mathbf{v} m, \tag{3b}$$

(equations (3.39) and (3.40) of [26b]); $m$ is the electromagnetic mass and $\gamma$ is the usual contraction factor in the SRT. Butler wrote: "...In the derivation of the Poynting's theorem *only the two curl equations were used.* The two divergence equations were ignored. Therefore Poynting's theorem is covariant only if both divergence terms are equal to zero!... only in the absence of charge! That is the reason that Pauli wrote..." Rohrlich indicated that the equations (3.39) and (3.40) are deduced from more general form (see (3.23) and (3.24) therein). The old definitions $U = \frac{1}{8\pi}(\mathbf{E}^2 + \mathbf{B}^2)$ and $\mathbf{S} = \frac{1}{4\pi}(\mathbf{E} \times \mathbf{B})$ are valid only when $\mathbf{v} \equiv 0$ (in fact this signifies that the old definitions are erroneous when applied to the electrodynamics with moving charges).[8] Unfortunately, the expression of Chubykalo [2, Eq.(28)] seems not to reduce to the Rohrlich-Butler formula in the steady-state limit (i.e. when $\mathbf{E}^* = \mathbf{H}^* = 0$).

Next, we shall try to deepen understanding the origins why did Chubykalo obtain such a strange result. We shall pay attention to the Lagrangian formalism [22,23]. In general, it is not clear if there may exist a situation when the fields does *not* tend to zero at the spatial

[8] See also the first paragraph on p. 1313 in [26b].

infinity, but their variation (and the variation of derivatives) do (the latter is necessary for the validity of the Lagrangian formalism). But, it is obvious that in the Chubykalo case one may wish to use the variation procedure for *two* types of potentials in order to try to obtain the formula (2). The ordinary Lagrangian

$$\mathcal{L} = -\frac{1}{2}\frac{\partial A_\mu}{\partial x^\nu}\frac{\partial A^\mu}{\partial x_\nu} \tag{4}$$

indispensably leads in the radiation gauge $A_0 = 0$ to the standard expression for the Hamiltonian: [28, p.147]

$$\mathcal{H} = \frac{1}{2}\int (|\mathbf{B}|^2 + |\mathbf{E}|^2) d^3x = \frac{1}{2}\int \left[|\nabla\times\mathbf{A}|^2 + \frac{1}{c}(\partial\mathbf{A}/\partial t)^2\right] d^3x\,. \tag{5}$$

The first way to think about the generalization of the Lagrangian for the Chubykalo case is to try to add the total 4-derivative[9] But, in seemingly appropriate case $\Gamma^\mu = A^\nu F^\mu{}_\nu$ we come to the situation when the second-order derivatives enter into the Lagrangian and the Ostrogradsky procedure becomes to be necessary.

Here, we suggest to generalize the Lagrangian for electromagnetic field described by the 4-vector potential in the following way:[10]

$$\mathcal{L} = -\frac{1}{4}\frac{\partial \mathcal{A}_\mu}{\partial x^\nu}\frac{\partial \mathcal{A}^\mu}{\partial x_\nu} - \frac{1}{4}\frac{\partial \overline{\mathcal{A}}_\mu}{\partial x^\nu}\frac{\partial \overline{\mathcal{A}}^\mu}{\partial x_\nu}\,. \tag{6}$$

Thus, we introduce the *complex* 4-potential $\mathcal{A}_\mu = A_\mu^{(elm)} + iA_\mu^*$, where $A_\mu^*$ corresponds to the fields $\mathbf{E}^*$ and $\mathbf{H}^*$ which do not vanish at the infinity. Obviously, it is Hermitian and of the first order in derivatives. If one varies over $A_\mu^{(elm)}$, the common-used potential, and $A_\mu^*$ independently (what, in fact, the Chubykalo-Smirnov-Rueda papers assumed), the new form of the Lagrangian (6)

---

[9]The Barut formula in ref. [22, p.107] (see the formula next to (3.62)) seems to be incorrect due to $\partial_\nu\Gamma^\nu$ may depend on the field derivatives and contribute to the second term of (3.62).

[10]We denote the complex conjugation by a line over a letter. This is used in order not to confuse with the "star" notation for the free fields $\mathbf{E}^*$ and $\mathbf{B}^*$ in [2].

$$\mathcal{L} = -\frac{1}{2}\frac{\partial A_\mu^{(elm)}}{\partial x^\nu}\frac{\partial A^{\mu\,(elm)}}{\partial x_\nu} + \frac{1}{2}\frac{\partial A_\mu^*}{\partial x^\nu}\frac{\partial A^{\mu\,*}}{\partial x_\nu} \tag{7}$$

leads to Eq. (2). So, this suggestion of ours may have some connections with ref. [15], where the idea of the *complex* 4-vector potential has been proposed. This idea should be compared with the Stepanovskiĭ work [29, p.189], which speculates on the unification of gravity with electromagnetism on the basis of complex potentials in the framework of the Weyl theory. [11]

Finally, I want to recall about the problem of energy in *quantum* electrodynamics. In ref. [23, p. 89] the book authors write *explicitly*: "*Quantization (12.2) [the standard one] does not provide the positive definiteness of the mean value of energy*" (sic!) After Gupta and Bleuler [31] by an artificial procedure ("by hands") they 1) assumed zero component of the 4-potential to be **anti-Hermitian** (sic!),[12] 2) indefinite metrics in the state-vectors space, and 3) the weak Lorentz condition. As a result, on the basis of the above postulates, they obtained the positive definiteness of the energy. If one wants to have negative and zero energies in the quantization of the electromagnetic field (as Chubykalo and Ahluwalia) the answer is simple: they should not apply additional postulates given few lines above.[13]

Finally, we explained the physical and mathematical content of the paper [2]. The conclusion is: *we are not yet convinced in the necessity of correction of the formula for the energy density for radiation field because of the absence of firm experimental and mathematical bases*

---

[11] The referee of *Foundation of Physics Letters* rejected [15a] on the basis of no-proof (in his opinion) of the idea of complex potentials. I disagreed with him on the basis of several papers exploring 1) possibility of various field operators (e.g., the negative energy part of a field operator can be not $C$ conjugate but $CP$ conjugate as in the case of neutrinos – other neutral particles); 2) complex-space coordinates; 3) possibility of presenting the spinorial affine connection as a complex 4-vector [30,29]. Furthermore, the concurrent paper [15b] has been published. I consider this referee to be biased and directed to the destruction of my work.

[12] This has again relevance to the idea of the complex 4-potential [15].

[13] Seems, the referee of *Foundation of Physics Letters* did not read the University textbooks.

*in [2].*

## ACKNOWLEDGMENTS

I greatly appreciate discussions with Profs. A. Chubykalo (1995-1999), patient answers on my questions by Profs. Y. S. Kim, M. Moshinsky and Yu. F. Smirnov, and the useful information from Profs. A. F. Pashkov (1982-99) and E. Recami (1995-99). Frank discussions with Prof. D. Ahluwalia (1993-98) are acknowledged, even if I do *not* always agree with him.

Zacatecas University, México, is thanked for awarding the full professorship. This work has been partly supported by the Mexican Sistema Nacional de Investigadores and the Programa de Apoyo a la Carrera Docente.

---

[1] S. Weinberg, *Phys. Rev.* **134**, B882 (1964).

[2] A. E. Chubykalo, *Mod. Phys. Lett.* A**13**, 2139 (1998).

[3] J. R. Oppenheimer, *Phys. Rev.* **38**, 725 (1931).

[4] R. H. Good, Jr., *Phys. Rev.* **105**, 1914 (1957) (see p. 1915); in "*Lectures in theoretical physics.* University of Colorado. Boulder" (Interscience, 1959), p. 30 (see p. 47); T. J. Nelson and R. H. Good, Jr., *Phys. Rev.* **179**, 1445 (1969) (see p. 1446).

[5] E. Gianetto, *Lett. Nuovo Cim.* **44**, No. 3, 140 (1985); ibid. 145 (1985). See pp. 142 and 147, respectively. E. Recami *et al.* were very close to re-discover this solution in the papers *Lett. Nuovo Cim.* **11**, 568 (1974) and in "*Hadronic mechanics and nonpotential interactions*", ed. M. Mijatovic, (Nova Science Pubs., New York, 1990), p. 231 but in the analysis of the determinant of the Maxwell equations they put additional constraint of transversality $\mathbf{p}\cdot\boldsymbol{\psi}=0$ on the solutions of Eq. (1).

[6] D. V. Ahluwalia and D. J. Ernst, *Mod. Phys. Lett.* **A7**, 1967 (1992); D. V. Ahluwalia, in "*Proceedings*

*of The Present Status of Quantum Theory of Light: A Symposium to Honour Jean-Pierre Vigier. York University, Toronto, Aug. 27-30, 1995*", eds. G. Hunter *et al.* (Kluwer Academic, 1996), p. 443.

[7] V. V. Dvoeglazov, Yu. N. Tyukhtyaev and S. V. Khudyakov, *Izv. VUZ: fiz.* **37**, No. 9, 110 (1994) [Translation: *Russ. Phys. J.* **37**, 898 (1994). This is a version of the Saratov University preprint of 1992, which was revised in order to include references to the works appeared in 1991-1994.

[8] M. Evans, *Found. Phys.* **24**, 1671 (1994). This paper started from other postulates, which may be correct. In spite of Dr. Evans' works contain some misunderstandings, he also indicated at the energy $E = 0$ associated with certain solutions of the Maxwell's equations (see the formula (41)).

[9] V. V. Dvoeglazov and M. Evans, *Apeiron* **4**, 45 (1997).

[10] V. V. Dvoeglazov, *Hadronic J. Suppl.* **12**, 241 (1997).

[11] M. Moshinsky and A. Del Sol, *Canadian J. Phys.* **72**, 453 (1994).

[12] W. S. Page and D. Cyganski, *"An example of the use of Maple in mathematical electrodynamics: The* $E = 0$ *solutions in "Toward an Understanding of Electromagnetic Phenomena" by Valeri Dvoeglazov and Myron W. Evans are not physically realizable."* Poster Session at *Causality and Locality in Modern Physics and Astronomy: Open Questions and Possible Solutions, Vigier-II Conference, Toronto, Canada, August 25-29, 1997.*

[13] S. Esposito, *Found. Phys.* **28**, 231 (1998).

[14] V. V. Dvoeglazov, *Normalization and* $m \to 0$ *limit of the Proca theory.* Preprint EFUAZ FT-96-39-REV, hep-th/9712036, Dec. 1997, accepted.

[15] V. V. Dvoeglazov, *On unusual interactions of the* $(1/2, 0) \oplus (0, 1/2)$ *particles.* Preprint EFUAZ FT-97-45-REV, hep-ph/9801287, Jan. 1998; S. Esposito, M. Evans and E. Recami, *Found. Phys. Lett.* **11**, 103 (1998), physics/9803040.

[16] W. A. Rodrigues and J. Vaz, jr., in "*The Theory of electron. Proceedings of the International Conference. México City, Sept. 24-27, 1995.*", eds. J. Keller and Z. Oziewicz (UNAM, México, 1997), pp. 457-466.

[17] "*Essays on the formal aspects of electromagnetic theory*" (Ed. A. Lakhtakia, World Scientific, 1993); "Advanced electromagnetism: foundations, theory and applications" (Eds. T. W. Barrett and D. M. Grimes, World Scientific, 1995).

[18] L. D. Landau and E. M. Lifshitz, *"The Classical theory of field"* (Moscow, Nauka, 1973), Russian edition.

[19] K. R. Brownstein, *Am. J. Phys.* **67**, 639 (1999).

[20] A. Chubykalo and R. Smirnov-Rueda, *Phys Rev.* E**53**, 5373 (1996); ibid. E**55**, 3793E (1997); *Mod. Phys. Lett.* A**12**, 1 (1997).

[21] P. A. M. Dirac, in *"Mathematical foundations of quantum theory"*, ed. by A.R. Marlow (Academic Press, Inc., 1978), p. 1; in "*Directions in physics*" ed. by H. Hora and J. R. Shepanski (John Wiley & Sons, New York). I particularly appreciate his approach to the science expressed in the quotation: "Any physical or philosophical ideas that one has must be adjusted to fit the mathematics. Not the other way around. Too many physicists are inclined to start from some preconceived physical ideas and then to try to develop them and find a mathematical scheme that incorporate them. Such a line of attack is unlikely to lead to success..."

[22] A. O. Barut, "*Electrodynamics and classical theory of fields and particles*" (Dover Publications, Inc., New York, 1979).

[23] N. N. Bogoliubov and D. V. Shirkov, "*Introduction into the theory of quantized fields*" (Moscow, Nauka, 1973), Russian edition.

[24] W. Pauli, "*Theory of relativity*" (Pergamon Press, Inc., New York, 1958), pp. 130 and 185.

[25] See, for instance, W. K. H. Panofsky and M. Phillips, "*Classical electricity and magnetism*" (Addison-Wesley Publ. Co., Reading, Mass., 1962), 2nd. edition, pp. 179, 377 and 378.

[26] F. Rohrlich, *Am. J. Phys.* **28**, 639 (1960); ibid. **38**, 1310 (1970); ibid. **64**, 16 (1996).

[27] J. W. Butler, *Am. J. Phys.* **36**, 936 (1968); ibid. **38**, 1258 (1970). See also P. Moylan, ibid. **63**, 818

(1995).

[28] L. H. Ryder, "*Quantum field theory*" (Cambridge University Press, Cambridge, 1985).

[29] Yu. P. Stepanovskiĭ, *Electromagnetic Phenomena* **1**, 180 (1998).

[30] H. Weyl, *Z. Phys.* B**56**, 330 (1929); *Sitzber. Kgl. Preuss. Acad. Wiss.*, p. 465 (1918); V. Bargmann, ibid., p. 346 (1932), cited in [29].

[31] S. Gupta, *Proc. Roy. Soc.* **63**A, 681 (1950); K. Bleuler, *Helv. Phys. Acta* **23**, 567 (1950).